

Sum of Numbers

$$1 + 2 + 3 + 4 = \frac{4}{2} (1+4) = 10$$

$$1+2+3+4+5+6 = \frac{6}{2} (1+6) = 3 \cdot 7 = 21$$

$$1+2+3+\dots+8+9+10 = \frac{10}{2} (1+10) = 5 \cdot 11 = 55$$

$$1+2+3+4+\dots+(n-2)+(n-1)+n = \frac{n}{2}(1+n)$$

$$1+2+3+4+\dots+(n-1)+n+(n+1)+(n+2)+\dots+2n-2+2n-1+2n = \frac{2n}{2} (1+2n)$$

Given $S_n = n^2 - 2n$

Find $\Rightarrow a_n = ? \quad a_1 = ? \quad a_2 = ? \quad a_3 = ?$

$$\begin{aligned} a_n &= S_n - S_{n-1} \quad ; \quad a_n = n^2 - 2n - \{(n-1)^2 - 2(n-1)\} = \\ &= n^2 - 2n - [n^2 - 2n + 1 - 2n + 2] = \\ &= -1 + 2n - 2 = 2n - 3 \end{aligned}$$

$$a_1 = 2 \cdot 1 - 3 = -1$$

$$a_2 = 2 \cdot 2 - 3 = 1$$

$$a_3 = 2 \cdot 3 - 3 = 3$$

we see that $(a_3 - a_2) = (a_2 - a_1)$