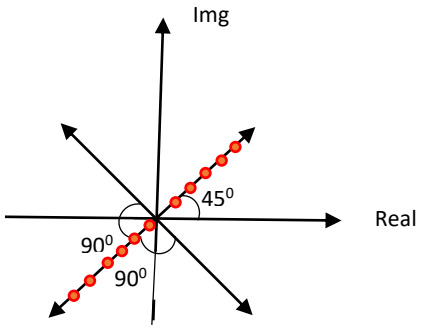
Complex Number $z^4 = -1$



$Z^4 = -1$, Find the roots

$Z^4 = \cos 180^0 + i\sin 180^0$

$Z^4 = \cos(180^0+360n) + i\sin(180^0+360n)$

Divide the angle by four (4)

$Z = \cos(45^0+90^0n) + i\sin(45^0+90^0n)$

(1) $Z = \cos 45^0 + i\sin 45^0$ $\quad \frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$

(2) $Z = \cos 135^0 + i\sin 135^0$ $\quad -\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$

(3) $Z = \cos 225^0 + i\sin 225^0$ $\quad -\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$

(4) $Z = \cos 315^0 + i\sin 315^0$ $\quad \frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$

Find the sum of roots:

$Z^3 = -1$

$Z^3 = \cos 180^0 + i\sin 180^0$

$Z_1 + Z_2 + Z_3 = (\frac{1}{2}+\frac{\sqrt{3}}{2}i) + (-1) + \frac{1}{2}-\frac{\sqrt{3}}{2}i = 0$