

Algebra Magic

$$(1) \frac{3}{x-1} - 1 = \frac{x-4}{5}, \quad x \neq 1$$

We know the answer

$$x = 4$$

$$x = -5 + 1 = -4,$$

$$\frac{3}{3} - 1 = 0, \quad \frac{3}{-5} - 1 = \frac{-8}{5}$$

$$(2) \frac{3}{x-1} - 1 = \frac{x-4}{2}, \quad x \neq 1$$

We know the answer, $x = 4$

$$x = -1$$

$$(3) \frac{3}{x-1} - 1 = \frac{x-4}{a}, \quad a \text{ is parameter}$$

$$x \neq 1$$

$$3a - a(x-1) = (x-1)(x-4)$$

$$x^2 + (a-5)x - 4(a-1) = 0$$

$$\Delta = (a-5)^2 + 16(a-1) = a^2 + 6a + 9 = (a+3)^2$$

$$x = \frac{5-a \pm (a+3)}{2}$$

$$x_1 = 4$$

$$x_2 = -a + 1$$