

2 Variables

$$x + y = 1$$

$$x^2 + y^2 = 2$$

$$x^3 + y^3 = ?$$

$$x + y = 1$$

$$y = 1 - x$$

$$x^2 + (1 - x)^2 - 2 = 0$$

$$2x^2 - 2x - 1 = 0$$

$$x = \frac{1 \pm \sqrt{1+2}}{2} = \frac{1 \pm \sqrt{3}}{2}$$

Example:

$$x = \frac{1+\sqrt{3}}{2} \Rightarrow y = 1 - \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) = \frac{1}{2} - \frac{\sqrt{3}}{2} = \frac{1-\sqrt{3}}{2}$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$= 1 \cdot (2 - xy)$$

$$x^3 + y^3 = 2 - xy$$

$$= 2 - \frac{1+\sqrt{3}}{2} \cdot \frac{1-\sqrt{3}}{2}$$

$$= 2 - \frac{1^2 - 3}{4}$$

$$= 2 - \left(\frac{-1}{2}\right) = 2 + \frac{1}{2} = 2\frac{1}{2}$$

$$x^3 + y^3 = 2\frac{1}{2}$$