

Limits

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{\sin(ax)}{x} &= \lim_{x \rightarrow 0} \frac{a \cdot \sin(ax)}{ax} = \\ &= a \cdot \lim_{x \rightarrow 0} \frac{\sin(ax)}{ax} = a \cdot 1 = a\end{aligned}$$

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{\sin(ax)}{3x} &= \frac{1}{3} \lim_{x \rightarrow 0} \frac{\sin(ax)}{x} = \\ &= \frac{1}{3} \lim_{x \rightarrow 0} \frac{a \cdot \sin(ax)}{ax} = \frac{a}{3} \lim_{x \rightarrow 0} \frac{\sin(ax)}{ax} = \frac{a}{3} \cdot 1 = \frac{a}{3}\end{aligned}$$

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{x - \sin x}{x} &= \lim_{x \rightarrow 0} \left(1 - \frac{\sin x}{x}\right) \\ 1 - \lim_{x \rightarrow 0} \frac{\sin x}{x} &= 1 - 1 = 0\end{aligned}$$

L'Hopital rule

$$\lim_{x \rightarrow 0} \frac{(x - \sin x)^1}{x^1} = \lim_{x \rightarrow 0} \frac{1 - \cos x}{1} = \frac{1-1}{1} = \frac{0}{1} = 0$$